Comoving suppression mechanism and cosmological constant problem

Jian Qi Shen*

Zhejiang Institute of Modern Physics and Department of Physics, Zhejiang University, Hangzhou 310027, P.R. China

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In this paper, we assume that the observer is fixed in a comoving frame of reference with $g_{00} = \frac{\lambda}{\Lambda}$, where λ and Λ denote the comoving parameter and the cosmological constant, respectively. By using the *comoving suppression mechanism* and *Mach's principle* (the latter of which is used to determine the comoving parameter λ), we calculate the vacuum energy density of quantum fluctuation field in the above-mentioned comoving frame of reference. It is shown that in such a comoving frame of reference, the cosmological constant will greatly decrease by many orders of magnitude (if Mach's principle is applied to this calculation, then it will be shown that Λ is reduced by about 120 orders of magnitude). Additionally, we briefly discuss the related topics such as the varying observed speed of light ($\frac{{\rm d}c}{{\rm d}t} = \mathcal{O}\left(10^{-9}{\rm m/s}^2\right)$) and the mystery of anomalous acceleration ($\sim 10^{-9}{\rm m/s}^2$) acquired by the Pioneer 10/11, Galileo and Ulysses spacecrafts.

Keywords: Comoving suppression mechanism, observed speed of light, cosmological constant

I. INTRODUCTION

Historically, the problem of vacuum energy density or cosmological constant [1] has been considered for many years. There are now two cosmological constant problems. One is that why the observed value of the vacuum energy density is so small (the ratio of experimental value to the theoretical one is only 10^{-120}). The other one is to understand why the observed vacuum energy density is not only small, but also, as current Type Ia supernova observations seem to indicate, of the same order of magnitude as the present mass density of the universe [2]. It is readily verified that the quantum vacuum fluctuation energy density of one field is $U = \frac{c^7}{8\pi^2\hbar G^2}$ and the cosmological constant is therefore $\Lambda = \frac{8\pi G}{c^4}U = \frac{c^3}{\pi\hbar G}$, which can be rewritten as $\Lambda_{\rm th} = \frac{1}{\pi}\frac{1}{r_{\rm Pl}^2}$, where the Planck length is $r_{\rm Pl} = \sqrt{\frac{G\hbar}{c^3}}$. However, it is also well known that the observed cosmological constant can be expressed[1] as $\Lambda_{\rm ex} = \frac{3H^2}{c^2} \simeq \frac{3}{r_{\rm Uni}^2}$, where $r_{\rm Uni}$ denotes the cosmological radius (length scale of the universe). So, the ratio of the theoretical cosmological constant, $\Lambda_{\rm th}$, to the observed one, $\Lambda_{\rm ex}$, is

$$\frac{\Lambda_{\rm th}}{\Lambda_{\rm ex}} = \frac{1}{3\pi}\left(\frac{r_{\rm Uni}}{r_{\rm Pl}}\right)^2 \simeq 10^{120}, \tag{1.1}$$

where the orders of magnitude of the Plank length and cosmological radius (*i.e.*, $r_{\rm Pl} \simeq 10^{-35}$ m and $r_{\rm Uni} \simeq 10^{26}$ m) have been inserted.

During the past 40 years, in an attempt to deal with the cosmological constant problem, many theoretical works such as adjustment mechanism, changing gravity, quantum cosmology and the viewpoint of supersymmetry, supergravity and superstrings were proposed [3]. More recently, since some astrophysical observations show that the large scale mean pressure of our present universe is negative suggesting a positive cosmological constant, and that the universe is therefore presently undergoing an accelerating expansion [4], a large number of theories and viewpoints (such as back reaction of cosmological perturbations [5], QCD trace anomaly [6], contribution of Kaluza-Klein modes to vacuum energy [7], five-dimensional unification of cosmological constant and photon mass [8], nonlocal quantum gravity [9], quantum micro structure of spacetime [10], relaxation of the cosmological constant in a movable brane world [11], the effect of minimal length uncertainty relation (under the modified commutation relation $[q, p]$) on the density of states [12], and so on) are put forward to resolve the cosmological constant problem. In this paper, we will suggest a so-called *comoving suppression mechanism* to resolve the cosmological constant problem. Differing from the conventional viewpoint that the observer is fixed at the comoving frame of reference with the metric $g_{00} = 1$, here we argue that the observer may be located in a comoving coordinate system of the metric $g_{00} = \frac{\lambda}{\Lambda}$ with λ being a certain parameter that will be discussed in the present paper. It will be shown that in such a comoving frame of reference the vacuum energy density can be greatly suppressed by the comoving compression mechanism. In order to determine the comoving parameter λ, we will make use of Mach's principle in the discussion.

The paper discuss three topics: (i) the concept of λ-de Sitter metric and the comoving parameter λ; (ii) the calculation of vacuum energy density in the λ- de Sitter universe; (iii) the varying observed speed of light and the potential relation to the anomalous acceleration ($\sim 10^{-9}{\rm m/s}^2$) acquired by the Pioneer 10/11, Galileo and Ulysses spacecrafts [13].

[1]This expression means that even though Λ is very small, it is comparable to the present matter density.

II. COMOVING FRAME OF REFERENCE AND λ- DE SITTER METRIC

In this section, first we propose the concept of λ- de Sitter world, the line element of which takes the form

$$\mathrm{d}s^2=\frac{\lambda}{\Lambda}c^2\mathrm{d}t^2-\exp\left(2\sqrt{\frac{\lambda}{3}}ct\right)\left[\mathrm{d}r^2+r^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2\right)\right], \tag{2.1}$$

which can be solved via Einstein field equation with the cosmological constant term. In this comoving frame of reference with $g_{00}=\frac{\lambda}{\Lambda}$ (where the coordinate time is no longer the proper time), the cosmological expansion rate equations are of the form

$$\begin{aligned}\frac{8\pi G}{c^2}\frac{\lambda}{\Lambda}\rho+\lambda&=\frac{\lambda}{\Lambda}\frac{3k}{R^2}+\frac{3\dot{R}^2}{c^2R^2},\\ \frac{8\pi G}{c^4}\frac{\lambda}{\Lambda}p&=\lambda-\frac{\lambda}{\Lambda}\frac{k}{R^2}-\frac{\dot{R}^2}{c^2R^2}-\frac{2\ddot{R}}{c^2R},\end{aligned} \tag{2.2}$$

where R denotes the cosmological scale factor. If we assume that the Hubble parameter $H=\frac{\dot{R}}{R}$ is constant and we can ignore the matter density and pressure[2] in Eq.(2.2), we may obtain $\lambda=3\frac{H^2}{c^2}$. In this step, someone may think that in Eq.(2.2) the comoving parameter λ acts just as the cosmological constant. This, however, may be not the true case, the reason for which will be considered as follows: the Hubble parameter H was measured by the Doppler redshift observation[3], where the redshift $z=\frac{\dot{R}}{R}\frac{L}{c\sqrt{\frac{\lambda}{\Lambda}}}$ with L being the physical distance between the observed star and the observer. Thus the observed Hubble parameter $H_{\rm obs}=\frac{\dot{R}}{R}=zc\sqrt{\frac{\lambda}{\Lambda}}\frac{1}{L}$. By using the above-obtained relation $\lambda=3\frac{H^2}{c^2}$, one can arrive at

$$\Lambda=\frac{3z^2c^2}{L^2}. \tag{2.3}$$

It should be noted that even though Eq.(2.3) is a familiar expression, it is of no help for overcoming the cosmological constant problem, since here the speed of light c is no longer the observed one. Instead, the observed speed of light is $c'=c\sqrt{\frac{\lambda}{\Lambda}}$ (rather than c). In the next section it will be verified that $\sqrt{\frac{\lambda}{\Lambda}}=10^{-30}$. This, therefore, implies that the fundamental constant c is 10^{30} times more than the observed speed of light ($c'=3.00\times10^8$ m/s). Thus it is believed that the above viewpoint cannot interpret the cosmological constant problem.

The previous discussion shows that the *observed* cosmological constant is not the physical meanings of λ, namely, λ cannot be considered a so-called *observed* cosmological constant. The *observed* cosmological constant is still Λ. In what follows we will consider the compression mechanism of vacuum energy density in the λ- de Sitter world (*i.e.*, the comoving frame of reference with $g_{00}=\frac{\lambda}{\Lambda}$).

III. SUPPRESSION OF Λ IN λ- DE SITTER WORLD

In this section, the vacuum energy density in the λ- de Sitter world will be calculated and, by taking account of Mach's principle, the reduction of Λ by 120 orders of magnitude will be demonstrated.

In the comoving coordinate system with $g_{00}=\frac{\lambda}{\Lambda}$, the action of a particle is $S=-mc\int\mathrm{d}s$ with

$$\mathrm{d}s=\sqrt{\frac{\lambda}{\Lambda}c^2\mathrm{d}t^2-\exp\left(2\sqrt{\frac{\lambda}{3}}ct\right)\mathrm{d}\mathbf{r}^2}=\sqrt{\frac{\lambda}{\Lambda}c^2-v^2}\mathrm{d}t, \tag{3.1}$$

where the test particle velocity squared is defined as $v^2=\exp\left(2\sqrt{\frac{\lambda}{3}}ct\right)\left(\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}\right)^2$. Thus the Lagrangian of the test particle under consideration is $L=-mc\sqrt{\frac{\lambda}{\Lambda}c^2-v^2}$. It follows that the canonical momentum and the canonical Hamiltonian read

$$p=\frac{\partial L}{\partial v}=\frac{mv}{\sqrt{\frac{\lambda}{\Lambda}-\frac{v^2}{c^2}}},\qquad H=pv-L=\frac{m\frac{\lambda}{\Lambda}c^2}{\sqrt{\frac{\lambda}{\Lambda}-\frac{v^2}{c^2}}}. \tag{3.2}$$

Let us define the observed mass and the observed speed of light as follows: $m'=\frac{m}{\sqrt{\frac{\lambda}{\Lambda}}}$, $c'=\sqrt{\frac{\lambda}{\Lambda}}c$. Clearly, the expressions (3.2) can be rewritten as

$$p=\frac{m'v}{\sqrt{1-\frac{v^2}{c'^2}}},\qquad H=\frac{m'c'^2}{\sqrt{1-\frac{v^2}{c'^2}}}, \tag{3.3}$$

which take the form familiar to us all. For the extremely relativistic case $v\to c'$, we have $pc'\to H$. I assume that the observers (say, we) are fixed actually in the comoving frame of reference with $g_{00}=\frac{\lambda}{\Lambda}$, and that the observed speed of light is c' rather than c. The calculated result for the vacuum energy density in the comoving frame of reference with $g_{00}=\frac{\lambda}{\Lambda}$ is

[2] Recent observation shows that the components of regular matter, dark matter and dark energy in the universe is as follows: 4% (regular matter), 23% (dark matter), 73% (dark energy). This, therefore, means that the component that is described by the cosmological constant term is more than $\frac{2}{3}$ of the total matter in the whole universe.

[3] The redshift is $z=\frac{\triangle\lambda}{\lambda}$ with λ being the wavelength of light.

$$U = \frac{2}{(2\pi\hbar)^3}\int^{p_{\rm Pl}} 4\pi p^2 \frac{1}{2}\hbar\omega_p {\rm d}p$$
$$\simeq \frac{2}{(2\pi\hbar)^3}\int^{p_{\rm Pl}} 4\pi p^2 \frac{1}{2}(pc'){\rm d}p, \qquad (3.4)$$

where the cutoff momentum is $p_{\rm Pl} = \sqrt{\frac{\hbar c'^3}{G}}$. Note that here the fundamental constant c has been replaced with the observed speed of light c' ($c' = 3.00 \times 10^8$ m/s). Further calculation yields $U = \frac{c'^7}{8\pi^2\hbar G^2}$, and consequently the calculated cosmological constant reads

$$\Lambda = \frac{8\pi G}{c^4}\frac{c'^7}{8\pi^2\hbar G^2}. \qquad (3.5)$$

It should be noted that in Eq.(3.5) the coefficient $\frac{8\pi G}{c^4}$ need not to be replaced with $\frac{8\pi G}{c'^4}$, since here c is a fundamental constant rather than an observed speed of light. By using the relation (*i.e.*, $c = \sqrt{\frac{\Lambda}{\lambda}}c'$) between c and c', Eq.(3.5) can be rewritten as $\Lambda^3 = \frac{c'^3}{\pi\hbar G}\lambda^2$. With the help of $r_{\rm Pl} = \sqrt{\frac{G\hbar}{c'^3}}$, one can arrive at

$$\Lambda = \frac{\lambda^{\frac{2}{3}}}{\pi^{\frac{1}{3}} r_{\rm Pl}^{\frac{2}{3}}}, \qquad (3.6)$$

which is expressed in terms of the Plank length and the comoving parameter λ. Thus it follows from Eq.(3.6) that the vacuum energy density (or cosmological constant Λ) is determined only by the comoving parameter λ. If λ is taken to be very small (or nearly vanishing), then the corresponding Λ is also very small. This is just the comoving suppression mechanism for the cosmological constant. Even though according to the conventional choice where it is assumed that the observer is fixed in the comoving frame of reference with $g_{00} = 1$, the vacuum energy density is very large (even nearly divergent), in the comoving frame with $g_{00} = \frac{\lambda}{\Lambda}$, Λ is greatly compressed by many orders of magnitude.

Now the problem left to us is to determine the value of the comoving parameter λ. Brief analysis indicates that it is in connection with Mach's principle: specifically, substitution of the observed cosmological constant $\Lambda \simeq \frac{3}{r_{\rm Uni}^2}$ into Eq.(3.6) yields

$$\lambda = \frac{r_{\rm Pl}}{r_{\rm Uni}^3}, \qquad \frac{\lambda}{\Lambda} \simeq \frac{r_{\rm Pl}}{r_{\rm Uni}} \sim 10^{-61}. \qquad (3.7)$$

How can we understand the physical meanings of the obtained λ in Eq.(3.7)? The expression $2\sqrt{\frac{\lambda}{3}}ct$ in the exponential factor of the line element (2.1) of the comoving frame with $g_{00} = \frac{\lambda}{\Lambda}$ reads

$$2\sqrt{\frac{\lambda}{3}}cT = \frac{2}{\sqrt{3}}\frac{c'T}{r_{\rm Uni}} = \mathcal{O}(1), \qquad (3.8)$$

where the expression for c, *i.e.*, $c = \frac{c'}{\sqrt{\frac{\lambda}{\Lambda}}} \sim 10^{38}$m/s has been inserted. In Eq.(3.8) T denotes the cosmological age of the present universe. In accordance with the result in Eq.(3.8), one can conclude without fear that the comoving parameter λ taking the form (3.7) just agrees with Mach's principle.

Additionally, it should be pointed out that the observed (and measured) mass m' of a particle is not the real mass m, the relation between which is given $m = \sqrt{\frac{\lambda}{\Lambda}}m' \simeq 10^{-30}m'$. It may be believed that this relation also has close relation to Mach's principle which holds that the inertial properties of a body are determined by the mass distribution in the universe, and the inertial force acting upon a body arises from an interaction between it and the matter-energy content of the whole universe. It is of physical interest that the real mass m is actually merely one part of 10^{30} of the observed one, which, therefore, means that the real total cosmological mass is only about 10^{23} Kg that is surprisingly less than that of the Earth (the observed mass of the Earth is 6.0×10^{24} Kg)!

In order to close this section, we briefly conclude the comoving suppression mechanism for the cosmological constant with some remarks. In view of the above discussion, the comoving suppression mechanism is established based on the following four points: (i) the assumption that the observer is fixed in a comoving frame of reference with $g_{00} = \frac{\lambda}{\Lambda}$; (ii) Mach's principle that can determine the comoving parameter λ; (iii) the observed speed of light $c' = c\sqrt{\frac{\lambda}{\Lambda}}$; (iv) the vacuum energy density $U = \frac{c'^7}{8\pi^2\hbar G^2}$ in the comoving frame of reference with $g_{00} = \frac{\lambda}{\Lambda}$. Thus the comoving suppression mechanism will inevitably lead to the relaxation of the cosmological constant by 120 orders of magnitude.

IV. THE VARYING OBSERVED SPEED OF LIGHT

According to the above comoving suppression mechanism, it is readily verified that the observed speed of light c' is not constant. In fact, its variation (rate of change) at present is

$$\frac{{\rm d}c}{{\rm d}t} = \mathcal{O}\left(10^{-9}{\rm m/s}^2\right). \qquad (4.1)$$

In 1998, Anderson *et al.* reported that, by ruling out a number of potential causes, radio metric data from the Pioneer 10/11, Galileo and Ulysses spacecraft indicate an apparent anomalous, constant, acceleration acting on the spacecraft with a magnitude $\sim 8.5\times 10^{-8}$cm/s^2 directed towards the Sun [13]. Is it the effects of dark matter or a modification of gravity? Unfortunately, neither easily works. It is interesting that by taking the cosmic mass,

$M = 10^{53}$ kg, and cosmic scale, $R = 10^{26}$ m, our calculation shows that *this acceleration is just equal to the value of field strength on the cosmic boundary due to the total cosmic mass.* This fact leads us to consider a theoretical mechanism to interpret this anomalous phenomenon. The author favors that the gravitational Meissner effect may serve as a possible interpretation. Here we give a rough analysis, which contains only the most important features rather than the precise details of this theoretical explanation. Parallel to London's electrodynamics of superconductivity, it shows that gravitational field may give rise to an *effective rest mass* $m_g = \frac{\hbar}{c^2}\sqrt{8\pi G\rho_m}$ due to the self-induced charge current [14], where ρ_m is the mass density of the universe. Then one can obtain that $\frac{\hbar}{m_g c} \simeq 10^{26}$m that approximately equals R, where the mass density of the universe is taken to be $\rho_m = 0.3\rho_c$ [15] with $\rho_c \simeq 2 \times 10^{-26}\mathrm{kg/m^3}$ being the critical mass density. An added constant acceleration, a, may result from the Yukawa potential and can be written as[4]

$$a = \frac{GM}{2}\left(\frac{m_g c}{\hbar}\right)^2 \simeq \frac{GM}{R^2} \simeq 10^{-9}\mathrm{m/s^2}. \tag{4.2}$$

Hence we demonstrated that the value of $10^{-9}\mathrm{m/s^2}$ is a very typical and interesting "acceleration", which arises in (i) the anomalous acceleration acting on the Pioneer 10/11, Galileo and Ulysses spacecrafts [13], (ii) gravitational Meissner effect [16], (iii) the gravitational field strength at the cosmic boundary, and (iv) even the varying observed speed of light. It is reasonably believed that these facts may clue physicists on the mystery of anomalous acceleration acquired by the Pioneer 10/11, Galileo and Ulysses spacecrafts [13]. Since this subject is beyond the scope of the present paper, we will not consider it further.

V. CONCLUDING REMARKS

The comoving suppression mechanism for the cosmological constant problem assumes that we (observers) are fixed in a comoving coordinate system with $g_{00} = \frac{\lambda}{\Lambda}$ rather than with $g_{00} = 1$, and that the comoving parameter λ can be determined by Mach's principle. In such a theoretical framework, the calculation of the vacuum energy density in this comoving frame of reference indicates that the cosmological constant is reduced by 120 orders of magnitude.

In the conventional comoving frame of reference, the metric of de-Sitter world is

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - \exp\left(2\sqrt{\frac{\Lambda}{3}}ct\right)\mathrm{d}\mathbf{r}^2 \tag{5.1}$$

rather than Eq.(2.1), *i.e.*,

$$\mathrm{d}s^2 = \frac{\lambda}{\Lambda}c^2\mathrm{d}t^2 - \exp\left(2\sqrt{\frac{\lambda}{3}}ct\right)\mathrm{d}\mathbf{r}^2. \tag{5.2}$$

It is believed that the comoving parameter λ in (5.2) possesses physical meanings and therefore deserves essential consideration. Note that since in Eq.(5.2), $\frac{\lambda}{\Lambda}c^2 = c'^2$ with $c' = 3.00 \times 10^8$ m/s, our experimental measurements (astrophysical observations) cannot demonstrate whether we are just in a comoving frame of reference with $g_{00} = \frac{\lambda}{\Lambda}$ rather than with $g_{00} = 1$, which, therefore, means that the comoving parameter λ should be determined on by Mach's principle. But, what is the physical origin of Mach's principle? This problem seems to be of no satisfactory solutions up to now. So, the key problem of the comoving suppression mechanism for the cosmological constant problem in the present paper may be the resolution of the physical meanings and origin of Mach's principle, which is now under consideration and will be published elsewhere.

[*]**Shen's electronic address**: jqshen@coer.zju.edu.cn

[4]It may also be calculated as follows: $a = \frac{GM}{2}\left(\frac{m_g c}{\hbar}\right)^2 = \frac{GM}{c^2}(4\pi\rho_m G) = \frac{GM}{c^2 R}\frac{G(4\pi R^3\rho_m)}{R^2} \simeq \frac{GM}{R^2}$, where use is made of $\frac{GM}{c^2 R} \simeq \mathcal{O}(1), 4\pi R^3\rho_m \simeq M$, which holds when the approximate evaluation is performed.

[1] on the history of the cosmological constant problem, see, for example, Straumann, N., (2002). gr-qc/0208027.
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